REPORT OF THE WAR PREPAREDNESS COMMITTEE OF THE AMERICAN MATHEMATICAL SOCIETY AND MATHEMATICAL ASSOCIATION OF AMERICA AT THE CHICAGO MEETING

This report is concerned with three important questions: (1) The deferment of mathematicians; (2) Report of Subcommittee on Education for Service; (3) Use of Selectees.

1. The Deferment of Mathematicians

When this problem arose for the first time over a year ago the great majority of mathematicians were against seeking any special consideration of mathematicians in connection with the Selective Service. It is the feeling of many that the situation has now changed to such an extent that steps should now be taken leading to the "individual occupational deferment" of mathematicians.

We are concerned in the first place with teachers of mathematics in the colleges and universities, with advanced students of mathematics, and with mathematicians in industry. Mathematicians in the industries essential for the defense are fairly well taken care of in accordance with recommendations to the Selective Service Boards by their employers. But teachers of mathematics are variously treated by the Selective Service Boards. Some men are deferred because they are teaching mathematics to engineers, but this rule is not general. A state Selective Service official to whom the case of a mathematician was appealed, stated that the national authorities had certified no shortage of mathematicians.

Because of the Selective Service law each man must be treated on his own merits in accordance with the principles of "individual occupational deferment." It became apparent on consultation with Dr. Zook of the American Council on Education and others that the opinions of mathematicians must be given statistical support. A subcommittee of the War Preparedness Committee on the Supply and Demand for Mathematicians was accordingly appointed in June as follows: Professors T. Y. Thomas (chairman), Harry Bateman, E. T. Bell, G. C. Evans, and W. M. Whyburn. This committee has begun its investigations. It has sent a questionnaire to the heads of departments of mathematics. The committee is concerned with the number, relative quality, induction, and employment of mathematicians of the Selective Service age. It will give special attention to the needs of engineering schools and industry.

If the results shown by this questionnaire warrant, a report should be made to General Hershey in charge of the Selective Service. Late last fall the American Council on Education issued a bulletin backed by General Hershey in which local Selective Service Boards were advised to give special consideration to men in chemistry, physics, engineering, biology, et cetera, with reference to the desirability of granting "individual occupational deferment." Mathematics was not listed. This bulletin was prepared with the aid of President Jewett of the National Academy.

This spring President Morse and Dean Richardson recommended to President Jewett that applied mathematics and, in particular, engineering mathematics be added to the fields listed in the bulletin of the American Council on Education.

In conclusion we may say that the opinion of mathematicians that there is a shortage or potential shortage of mathematicians will not affect the Selective Service policy unless backed by an established statistical position and a united front.

2. Subcommittee on Education for Service

An excellent report by this committee has been published in the American Mathematical Monthly. The earlier joint report with Morse was published in the Monthly and in the Mathematics Teacher. Copies were bought by the Government in some numbers. Considerable interest has been shown by various schools and teachers associations. The Hart report has reached a large number of college and university teachers of mathematics and some high school teachers. Nevertheless it is felt desirable that it also be brought to the attention of administrators of secondary schools.[1]

3. On the Use of Selectees

A considerable number of young mathematicians write to Morse as Chairman of the War Preparedness Committee stating that they may be selected and asking for help. Some wish to be used outside the Selective Service, others merely to be used in the Selective Service in some way appropriate to their mathematical training.

Very little can be done to obtain employment for these men outside the Selective Service. However, in all cases a letter has been sent to the Adjutant General giving essential data concerning the selectee together with recommendations as to his use when he is especially well qualified. These letters are cordially received by the Adjutant Gen-

[1] The Council and Board of Governors subsequently voted to distribute the report to the list of 3500 superintendents of schools of the National Education Association.

eral who has prescribed the general nature of the data desired. These data include a brief tabular vita, with age, degrees, positions, experience of mathematician and number and address of local Selective Service Board. Courses in engineering and experience with precision instruments are important. A number of the better trained young mathematicians have been placed in positions where their talents will be used to advantage.

MARSTON MORSE,
Chairman, War Preparedness Committee

THE SUMMER MEETING IN CHICAGO

The forty-seventh Summer Meeting of the Society and the twenty-third Colloquium were held at the University of Chicago, Chicago, Illinois, Tuesday to Saturday, September 2–6, 1941. The Mathematical Association of America met on Monday morning and afternoon, and jointly with the Society on Wednesday afternoon. The Institute of Mathematical Statistics held sessions Tuesday morning and afternoon and Wednesday afternoon. The Econometric Society met Tuesday, Wednesday and Thursday mornings and Tuesday afternoon.

The meeting was held in connection with the Fiftieth Anniversary Celebration of the University of Chicago, and on Tuesday morning Mr. Frederic Woodward, vice president emeritus of the University and director of the Celebration, gave a greeting and welcome to the mathematicians. As a part of the Fiftieth Anniversary Celebration, the University of Chicago collaborated with the Society in arranging a special program of lectures and conferences.

The number of papers presented at this meeting is the largest in the history of the Society and the attendance the second largest. Six hundred thirty-five persons registered including the following four hundred twenty-four members of the Society:

C. R. Adams, V. W. Adkisson, R. P. Agnew, A. A. Albert, G. E. Albert, C. B. Allendoerfer, Warren Ambrose, M. R. Anderson, T. W. Anderson, K. J. Arnold, Emil Artin, W. L. Ayres, H. M. Bacon, R. M. Ballard, J. D. Bankier, R. W. Barnard, I. A. Barnett, Walter Bartky, R. E. Basye, Harry Bateman, R. A. Beaumont, M. M. Beenken, A. A. Bennett, D. L. Bernstein, R. M. Besancon, E. E. Betz, H. R. Beveridge, S. F. Bibb, Garrett Birkhoff, G. D. Birkhoff, H. L. Black, D. H. Blackwell, H. F. Blichfeldt, G. A. Bliss, Henry Blumberg, L. M. Blumenthal, M. L. Boas, R. P. Boas, Salomon Bochner, D. G. Bourgin, J. W. Bower, M. G. Boyce, A. T. Brauer, Richard Brauer, H. E. Bray, R. W. Brink, Leonard Bristow, F. L. Brooks, H. E. Buchanan, C. E. Bures, F. J. H. Burkett, Herbert Busemann, Jewell H. Bushey, J. H. Bushey, W. H. Bussey, W. D. Cairns, J. W. Calkin, R. H. Cameron, C. C. Camp, E. J. Camp, H. H. Campaigne, A. D. Campbell, J. W. Campbell, W. B. Carver, C. R. Cassity, W. E. Cederberg, E. W. Chittenden, C. E. Clark, E. H. Clarke, M. D. Clement, A. B. Coble, C. J. Coe, L. M. Coffin, J. B. Coleman, Max Coral, J. J. Corliss, A. T. Craig, C. C. Craig, G. F. Cramer, H. B. Curry, H. B. Curtis, D. R. Curtiss, J. H. Curtiss, J. F. Daly, D. R. Davis, W. M. Davis, M. M. Day, J. J. DeCicco, L. J. Deck, F. F. Decker, C. H. Denbow, W. W. Denton, L. E. Dickson, Bernard Dimsdale, L. L. Dines, J. M. Dobbie, J. L. Doob, H. P. Doole, Arnold Dresden, D. M. Dribin, R. J. Duffin, Nelson Dunford, W. D. Duthie, P. S. Dwyer, J. J. Eachus, J. M. Earl, E. D. Eaves, M. C. Eide, Samuel Eilenberg, Churchill Eisenhart, M. L. Elveback, Benjamin Epstein, Paul Erdös, W. S. Erickson, H. J. Ettlinger, H. P. Evans, C. J. Everett, H. S. Everett, G. M. Ewing, F. A. Ficken, B. F. Finkel, C. H. Fischer, Edward Fleisher, L. R. Ford, R. M. Foster, R. H. Fox, Bernard Friedman, R. E. Fullerton, M. G. Gaba, E. R. Garbe, B. H. Gere, B. C. Getchell, K. S. Ghent, J. H. Giese, J. W. Givens, A. M. Gleason, H. H. Gold-

stine, Michael Golomb, Cornelius Gouwens, C. H. Graves, L. M. Graves, W. L. Graves, F. L. Griffin, V. G. Grove, P. E. Guenther, P. R. Halmos, R. W. Hamming, E. H. Hanson, O. G. Harrold, W. L. Hart, M. L. Hartung, J. O. Hassler, Camilla Hayden, N. A. Haynes, K. E. Hazard, E. R. Hedrick, M. H. Heins, E. D. Hellinger, M. J. Herzberger, Fritz Herzog, A. D. Hestenes, M. R. Hestenes, J. F. Heyda, J. S. Hickman, T. H. Hildebrandt, F. C. Hinds, J. J. L. Hinrichsen, G. P. Hochschild, F. E. Hohn, D. L. Holl, M. P. Hollcroft, T. R. Hollcroft, A. S. Householder, E. M. Hove, E. M. Hull, M. G. Humphreys, E. V. Huntington, D. H. Hyers, M. H. Ingraham, Dunham Jackson, R. L. Jackson, Nathan Jacobson, R. D. James, R. L. Jeffery, E. D. Jenkins, L. L. Johnson, M. M. Johnson, F. B. Jones, H. A. Jordan, Mark Kac, Shizuo Kakutani, G. K. Kalisch, Wilfred Kaplan, Irving Kaplansky, William Karush, D. E. Kearney, I. F. Keeler, J. L. Kelley, Claribel Kendall, J. F. Kenney, J. R. F. Kent, R. B. Kershner, P. W. Ketchum, J. M. Kinney, Fred Kiokemeister, C. J. Kirchen, J. R. Kline, L. A. Knowler, E. R. Kolchin, Horace Komm, C. F. Kossack, W. C. Krathwohl, M. E. Ladue, W. D. Lambert, O. E. Lancaster, E. P. Lane, R. E. Lane, R. E. Langer, C. G. Latimer, D. H. Leavens, Solomon Lefschetz, R. A. Leibler, Walter Leighton, B. A. Lengyel, C. A. Lester, P. E. Lewis, C. B. Lindquist, Marie Litzinger, Charles Loewner, M. I. Logsdon, L. A. Lorch, R. G. Lubben, C. I. Lubin, N. H. McCoy, E. A. McDougle, A. W. McGaughey, J. V. McKelvey, M. M. McKelvey, Brockway McMillan, E. J. McShane, Saunders MacLane, H. M. MacNeille, H. F. MacNeish, W. G. Madow, J. D. Mancill, Szolem Mandelbrojt, Morris Marden, C. W. Mathews, J. R. Mayor, A. E. Meder, Karl Menger, Herman Meyer, H. L. Meyer, D. D. Miller, E. W. Miller, W. L. Miser, M. G. Moore, R. L. Moore, T. W. Moore, W. L. Moore, C. W. Moran, Vladimir Morkovin, Richard Morris, Marston Morse, E. J. Moulton, D. C. Murdoch, F. H. Murray, S. B. Myers, J. P. Nash, A. L. Nelson, Albert Neuhaus, John von Neumann, C. V. Newsom, K. L. Nielsen, Ivan Niven, E. A. Nordhaus, E. B. Ogden, Rufus Oldenburger, E. G. Olds, J. M. H. Olmsted, E. J. Olson, Isaac Opatowski, Oystein Ore, F. W. Owens, H. B. Owens, J. C. Oxtoby, Gordon Pall, T. P. Palmer, R. S. Park, R. S. Pate, J. F. Paydon, C. K. Payne, A. M. Pegram, I. E. Perlin, Sam Perlis, J. W. Peters, O. J. Peterson, H. R. Phalen, R. S. Phillips, C. G. Phipps, T. A. Pierce, Everett Pitcher, George Polya, Hillel Poritsky, J. E. Powell, G. B. Price, W. T. Puckett, H. A. Rademacher, Tibor Radó, W. C. Randels, R. B. Rasmusen, Maxwell Reade, Francis Regan, P. V. Reichelderfer, W. T. Reid, Haim Reingold, Eric Reissner, C. E. Rickart, J. F. Ritt, C. V. Robinson, S. L. Robinson, V. N. Robinson, W. H. Roever, C. F. Roos, J. B. Rosenbach, R. A. Rosenbaum, Arthur Rosenthal, A. E. Ross, R. G. Sanger, Max Sasuly, A. C. Schaeffer, Henry Scheffé, M. A. Scheier, Peter Scherk, O. F. G. Schilling, Abraham Schwartz, H. M. Schwartz, G. E. Schweigert, I. E. Segal, M. E. Shanks, J. W. Sheedy, Seymour Sherman, C. G. Shover, D. R. Shreve, Harry Siller, H. A. Simmons, M. E. Sinclair, M. M. Slotnick, M. F. Smiley, A. H. Smith, E. R. Smith, F. C. Smith, H. L. Smith, R. G. Smith, Virgil Snyder, W. S. Snyder, R. H. Sorgenfrey, T. H. Southard, V. E. Spencer, A. A. Stafford, G. W. Starcher, M. E. Stark, N. E. Steenrod, B. M. Stewart, M. H. Stone, E. C. Stopher, E. B. Stouffer, W. J. Strange, Otto Szász, Gabor Szegö, A. H. Taub, M. E. Taylor, W. C. Taylor, J. M. Thompson, R. M. Thrall, W. J. Thron, L. W. Tordella, W. J. Trjitzinsky, P. L. Trump, A. W. Tucker, J. W. Tukey, A. E. Turner, S. M. Ulam, E. H. Umberger, Henry Van Engen, J. I. Vass, W. A. Vezeau, R. W. Wagner, Abraham Wald, R. J. Walker, Henry Wallman, R. M. Walter, T. O. Walton, J. H. Weaver, M. S. Webster, J. V. Wehausen, M. J. Weiss, B. A. Welch, C. P. Wells, E. T. Welmers, I. L. Wente, M. E. Wescott, F. J. Weyl, Hermann Weyl,

G. W. Whitehead, A. L. Whiteman, E. A. Whitman, P. M. Whitman, W. F. Whitmore, G. T. Whyburn, D. V. Widder, Norbert Wiener, L. R. Wilcox, R. L. Wilder, S. S. Wilks, E. W. Wilson, František Wolf, F. E. Wood, E. R. Worthington, C. H. Yeaton, J. W. T. Youngs, Oscar Zariski, J. W. Zimmer, Antoni Zygmund.

The Colloquium Lectures on the subject *Mathematical relations and structures* were given by Professor Oystein Ore of Yale University Wednesday, Thursday and Friday mornings. The presiding officers were Professors A. A. Albert, C. G. Latimer, and Emil Artin, respectively.

The sixteenth Josiah Willard Gibbs Lecture was given by Professor Sewall Wright of the University of Chicago on Wednesday evening. His title was *Statistical genetics and evolution.* President Marston Morse presided at this lecture.

Four general lectures were given under the composite title *Trends in research.* These were: *Abstracts spaces* by Professor M. H. Stone, Tuesday morning; *Topology* by Professor Solomon Lefschetz, Tuesday afternoon; *The calculus of variations* by Professor G. A. Bliss, Wednesday afternoon; and *The analytic theory of numbers* by Professor H. A. Rademacher, Friday afternoon. The presiding officers for these lectures were, in order, President Marston Morse, and Professors R. L. Wilder, R. W. Brink, and L. E. Dickson.

To present and discuss the latest developments in the fields of Algebra and the Theory of Integration, Conferences were held in these two subjects. These met simultaneously on Wednesday, Thursday and Saturday mornings. In the Conference on Algebra the speakers with their subjects were: Professor Garrett Birkhoff, *Lattice-ordered abelian groups*; Professor John von Neumann, *Complete lattice abelian groups*; Professor N. H. McCoy, *Some aspects of the theory of matrices over a ring*; Professor Nathan Jacobson, *Lie algebras*; Professor J. F. Ritt, *Differential polynomials*; Professor Oscar Zariski, *Some recent results in the arithmetical theory of algebraic varieties*; and Dr. O. F. G. Schilling, *A Hilbert series theory for general algebraic extensions.* The presiding officers of the Conference on Algebra were, in order, Professors A. A. Albert, Richard Brauer, and Hermann Weyl. In the Conference on the Theory of Integration the lectures were: Professor Salomon Bochner, *Integration and partial ordering*; Professor G. B. Price, *Integration and convex operators*; Professor T. H. Hildebrandt, *The general integrals of E. H. Moore*; Professor R. L. Jeffery, *Some recent advances in the theory of non-absolutely convergent integrals*; Professor Norbert Wiener, *Measure in statistical mechanics*; Professor Garrett Birkhoff, *Product integrals*; and Professor Nelson Dunford, *Applications of the theory of integration to the study of linear operators.* The

presiding officers for the three sessions of the Conference on the Theory of Integration were Professors L. M. Graves, John von Neumann, and E. J. McShane.

Thirteen sectional sessions for the reading of short papers were held as follows: Mathematical Statistics (joint session with the Institute of Mathematical Statistics and Econometric Society), Tuesday morning; Analysis, Tuesday morning, Tuesday afternoon (two sections), Friday morning, Friday afternoon; Geometry, Tuesday morning; Topology and the Theory of Sets, Tuesday morning; Algebra, Tuesday afternoon, Friday afternoon; Algebra and the Theory of Numbers, Friday morning; Topology, Friday morning; Applied Mathematics, Friday afternoon. The presiding officers for these sectional sessions were, in order, Professors A. T. Craig, D. R. Curtiss, Dunham Jackson, Salomon Bochner, C. R. Adams, T. H. Hildebrandt, Virgil Snyder, J. W. Tukey, A. A. Bennett, Saunders MacLane, M. H. Ingraham, Solomon Lefschetz, and Norbert Wiener.

Headquarters for the meeting were in Judson Court. The dormitories of the University of Chicago were available to the members of the four mathematical organizations and their guests. The general lectures and the Gibbs Lecture were held in Kent Chemical Laboratory. All other sessions met in Eckhart Hall and the adjoining Ryerson Physical Laboratory.

On Monday evening the members of the Department of Mathematics of the University of Chicago gave an informal reception in Burton Court Lounge. Tea was served in Eckhart Hall on Tuesday and Wednesday afternoons by the ladies of the Department. There was a bridge tournament on Tuesday evening. A luncheon and fashion show were held at Marshall Field and Company on Friday, and Friday evening a visit was made to Station WGN. Thursday afternoon was left free for recreation.

On Thursday evening there was a brief reception with officers of the four organizations receiving. This was followed by the joint dinner in Hutchinson Commons with an attendance of three hundred forty-one. President Marston Morse presided at the dinner. Professor Arnold Dresden spoke of the founding and early days of mathematics at the University of Chicago and particularly of the triumvirate Moore-Bolza-Maschke. At the close of his address he introduced the following resolution of greeting to Professor Oskar Bolza, the sole surviving member of this trio, which was adopted by a unanimous vote:

> A large group of members of the American Mathematical Society and their guests, meeting in Chicago to take part in the Celebra-

tion of the Fiftieth Anniversary of the Founding of the University of Chicago, send you their most affectionate greetings, mindful of the great contribution you have made to the development of mathematics in America and to its growth at the University of Chicago.

Professor Dresden was followed by Professor S. S. Wilks, who told of the history and purpose of the Institute of Mathematical Statistics and the Econometric Society, Professor D. R. Curtiss, who spoke of student days in Paris and the French professors of this era, and Professor G. A. Bliss, who read an announcement of the National Science Fund.

At the close of the dinner a resolution, introduced by Professor F. L. Griffin, extending felicitations to the University of Chicago upon its semicentennial and thanking it for the privilege of meeting as a part of the Celebration and expressing thanks to the Department of Mathematics for its fine arrangements and gracious hospitality, was adopted by a unanimous rising vote.

There were two meetings of the Council, one being a joint session with the Board of Governors of the Mathematical Association of America. This joint session to hear the report of the War Preparedness Committee was held at 4:30 P.M. in Eckhart Hall on Monday. This report appears elsewhere in this issue of the Bulletin. The other meeting of the Council was held at 8:15 P.M. on Tuesday in Judson Court Library.

The Secretary announced the election of the following twenty-nine persons to membership in the Society:

Dr. Fred Assadourian, New York University;
Dr. Herman Russell Branson, Dillard University, New Orleans, La.;
Dr. George William Brown, R. H. Macy and Company, New York, N. Y.;
Mr. George Eaddie Bulloch, New Jersey Bell Telephone Company, Newark, N. J.;
Professor Elías Alfredo de Cesare, University of La Plata;
Dr. Myron Abraham Coler, Paragon Paint and Varnish Corporation, Long Island City, N. Y.;
Mr. Alexander Robert Craw, University of Notre Dame;
Dr. Joseph Francis Daly, Catholic University of America;
Professor Max Wilhelm Dehn, University of Idaho;
Mr. Jacques Dutka, Bronx, New York, N. Y.;
Mr. E. LaVerne Eagle, Dayton Y.M.C.A. College, Dayton, Ohio;
Professor Laurence Hadley, Purdue University;
Professor Louise Stokes Hunter, Virginia State College;
Professor Alfred Edward Johns, McMaster University, Hamilton, Ontario, Canada;
Mr. Seymour Matthew Kwerel, Columbia University;
Mr. Thorstein Larsen, Consolidated Edison Company, Brooklyn, N. Y.;
Dr. Charles Loewner, University of Louisville;

Dr. Robert Sewell Pate, University of Illinois;
Mr. Murray Harold Protter, Brown University;
Professor Frank Atkinson Rickey, Louisiana State University;
Professor Mario O. González Rodríguez, University of Havana;
Dr. Leonard Jimmie Savage, Institute for Advanced Study;
Dr. William John Schart, Ohio State University;
Dr. Joseph A. Sharpe, Stanolind Oil and Gas Company, Tulsa, Okla.;
Dr. Ronald William Shephard, University of California;
Mr. William Henry Smith, Jr., Washington, D. C.;
Mr. Jonathan Dean Swift, California Institute of Technology;
Dr. Leopoldo Toralballa, Government Service Insurance System, Manila, Philippine Islands;
Dr. František Wolf, Macalester College, St. Paul, Minn.

It was reported that President Marston Morse will represent the Society at the One Hundred and Seventy-fifth Anniversary of the Founding of Rutgers University on October 9, 10, and 11, 1941. The following appointment by President Morse was also reported: as a subcommittee of the War Preparedness Committee, to be known as the Committee on Supply and Demand for Mathematicians, Professors T. Y. Thomas (chairman), Harry Bateman, E. T. Bell, G. C. Evans, and W. M. Whyburn.

Reports were sent in August to the Carnegie Corporation and Rockefeller Foundation regarding their grants to Mathematical Reviews.

The Secretary reported that copies of the recent report entitled *Industrial Mathematics* by Dr. T. C. Fry were distributed in August to members of the Society who are not members of the Mathematical Association of America.

Certain invitations to give hour addresses were announced: Professor G. B. Price for the November 1941 meeting in Manhattan, Kansas; Professors L. V. Bewley, I. S. Sokolnikoff and Oscar Zariski for the 1941 Annual Meeting at Lehigh University; Professor Norman Levinson for the February 1942 meeting in New York; Professor L. M. Blumenthal for the April 1942 meeting in Chicago; and Professor W. L. Ayres for the 1942 Summer Meeting at Cornell University.

On recommendation of the Committee on Places of Meetings (Professor A. B. Coble, chairman), the eastern spring meeting of 1942 was set at New York City, April 3–4. Times and places of meetings during 1942 were also set as follows: in New York City on February 28; in Berkeley, California on April 11; in Chicago on April 17–18; at Cornell University on September 8–11; in New York City on October 31; at the University of Notre Dame on November 27–28.

The seventeenth Josiah Willard Gibbs Lecture is to be given by

Professor John von Neumann at the Annual Meeting of 1942 in New York City.

Dean L. P. Eisenhart and Dean R. G. D. Richardson were appointed representatives on the Council of the American Association for the Advancement of Science for 1942 and Professor M. H. Stone was appointed representative on the National Research Council for a period of three years beginning July 1942.

The committee on the proposed new series of monographs, Mathematical Surveys (Professor J. L. Walsh, chairman), was empowered to proceed with the proposal presented by the Williams and Wilkins Company. In accordance with this proposal, the Company has agreed to publish at least two monographs, without commitment by the Society or the Company as to continuation. This committee was also empowered to appoint a temporary editorial board to act in connection with arrangements for the two monographs.

Professor G. B. Price reported for the Committee on Circulation for Mathematical Reviews that the total mailing list had increased from 1,222 on July 1, 1940 to 1,369 on July 1, 1941. He also stated that new subscriptions are continuing to arrive from China, Japan, Russia, England, et cetera, thus emphasizing the truly international character of this journal.

A committee consisting of Professors M. H. Ingraham (chairman), L. M. Graves, and J. W. Tukey was appointed to consider the question of dues for persons drafted or enlisted for military service.

The Council voted to recommend to the Society that Section 9, Article VII of the By-Laws be amended so that no new life members will be accepted. Action will be taken on this proposal at the October meeting in New York City.

Titles and cross references to the abstracts of the papers read at this meeting follow below. Papers numbered 1 to 6 were read before the section for Mathematical Statistics, papers 7 to 12 before the section for Analysis Tuesday morning, papers 13 to 19 before the section for Geometry, papers 20 to 24 before the section for Topology and the Theory of Sets, papers 25 to 28 before the first section for Analysis Tuesday afternoon, papers 29 to 33 before the section for Algebra Tuesday afternoon, papers 34 to 38 before the second section for Analysis Tuesday afternoon, papers 39 to 45 before the section for Analysis Friday morning, papers 46 to 51 before the section for Algebra and the Theory of Numbers, papers 52 to 59 before the section for Topology, papers 60 to 66 before the section for Analysis Friday afternoon, papers 67 to 72 before the section for Algebra Friday afternoon, and papers 73 to 76 before the section for Applied

Mathematics. Papers 77 to 141, whose abstract numbers are followed by the letter *t*, were read by title. Paper number 9 was read by Professor Schaeffer, 10 by Dr. Bankier, 11 by Mr. Thron, 29 by Dr. Pate, 40 by Dr. Duffin, 58 by Professor Eilenberg, 60 by Dr. Kac, 61 by Dr. Halmos, and 62 by Professor Kakutani. Professor Burr was introduced by Professor W. L. Ayres, Dr. Mood by Professor E. G. Olds, Mr. Allen by Dr. P. E. Lewis, Mr. Young by Professor R. L. Moore, Professor Dehn by Professor Oswald Veblen, Dr. Weinstein by Dr. H. E. Robbins, and Dr. Nicholson by Professor S. E. Warschawski.

1. J. B. Coleman: *A geometric derivation of Fisher's z-transformation.* (Abstract 47-9-433.)

2. Abraham Wald: *Large sample distribution of the likelihood ratio.* (Abstract 47-11-486.)

3. I. W. Burr: *Cumulative frequency functions.* (Abstract 47-9-432.)

4. K. J. Arnold: *On spherical probability distributions.* (Abstract 47-9-431.)

5. A. M. Mood: *On the asymptotic distribution of medians of samples from a multivariate population.* (Abstract 47-9-436.)

6. J. F. Daly: *A problem in estimation.* (Abstract 47-11-483.)

7. L. R. Ford: *The proper fractions.* (Abstract 47-9-379.)

8. Otto Szász: *Some new summability methods with applications.* (Abstract 47-9-404.)

9. R. J. Duffin and A. C. Schaeffer: *Power series with bounded coefficients.* (Abstract 47-9-376.)

10. J. D. Bankier and Walter Leighton: *Arithmetic continued fractions.* (Abstract 47-9-366.)

11. Walter Leighton and W. J. Thron: *Convergence criteria for continued fractions.* (Abstract 47-7-308.)

12. J. F. Paydon: *Convergence regions and value regions for continued fractions.* Preliminary report. (Abstract 47-11-473.)

13. Abraham Schwartz: *A consequence of the Ricci equations for riemannian manifolds.* (Abstract 47-9-426.)

14. T. R. Hollcroft: *Types of distinct double points of primals.* (Abstract 47-9-423.)

15. V. N. Robinson: *A transformation between osculating curves of a rational normal curve in an odd dimensional space.* (Abstract 47-7-323.)

16. J. J. DeCicco: *Geometric characterization of functions of n complex variables.* (Abstract 47-9-420.)

17. F. E. Hohn: *On Cartan's projective involutes.* (Abstract 47-9-422.)

18. J. H. Giese: *Conformally flat hypersurfaces of spaces of constant curvature.* (Abstract 47-9-421.)

19. E. F. Allen: *On a triangle inscribed in a rectangular hyperbola.* (Abstract 47-9-417.)

20. R. G. Lubben: *Decompositions of point sets and of portions of spaces.* (Abstract 47-9-448.)

21. Horace Komm: *Partial orders in euclidean spaces.* (Abstract 47-9-446.)

22. S. M. Ulam: *Theory of the operation of products of sets.* I. Preliminary report. (Abstract 47-9-406.)

23. F. A. Ficken: *Closure-mappings of partially ordered sets.* Preliminary report. (Abstract 47-9-441.)

24. Henry Blumberg: *A reconsideration of the paradoxes in the theory of sets.* (Abstract 47-9-429.)

25. R. H. Cameron: *Quadratic convolution equations.* (Abstract 47-9-370.)

26. R. E. Langer: *A theory for ordinary linear differential boundary problems of highly irregular type.* (Abstract 47-9-391.)

27. I. E. Perlin: *A calculus of variations problem with end points as functions of the curve.* Preliminary report. (Abstract 47-9-398.)

28. František Wolf: *On generalized boundary values and the uniqueness of corresponding Dirichlet problems.* (Abstract 47-9-407.)

29. R. J. Duffin and R. S. Pate: *Generalized groups and the Jordan-Hölder theorem.* (Abstract 47-9-340.)

30. Morris Marden: *The zeros of certain composite polynomials.* (Abstract 47-9-345.)

31. H. H. Campaigne: *An example of a cogroup.* (Abstract 47-9-337.)

32. D. C. Murdoch: *Automorphisms and quasi-groups.* Preliminary report. (Abstract 47-9-347.)

33. Rufus Oldenburger: *The factorization of polynomials.* (Abstract 47-9-349.)

34. D. G. Bourgin: *Some properties of Banach spaces.* (Abstract 47-9-369.)

35. F. B. Jones: *Measure and other properties of a Hamel basis.* (Abstract 47-9-386.)

36. R. S. Phillips: *On weakly compact subsets of a Banach space.* Preliminary report. (Abstract 47-9-399.)

37. M. M. Day: *Reflexivity criteria for a Banach space.* (Abstract 47-9-374.)

38. Herman Meyer: *Polynomial approximations to functions defined on abstract spaces.* (Abstract 47-9-395.)

39. Bernard Friedman: *Fourier coefficients of odd functions.* (Abstract 47-9-380.)

40. R. J. Duffin and J. J. Eachus: *A Paley-Wiener type expansion theorem.* (Abstract 47-9-375.)

41. R. P. Boas: *Generalized Laplace integrals.* (Abstract 47-7-303.)

42. R. P. Agnew: *Analytic extension by Hausdorff methods.* (Abstract 47-9-363.)

43. J. H. Curtiss: *Riemann sums and the fundamental polynomials of Lagrange interpolation.* (Abstract 47-7-305.)

44. J. M. Earl: *Polynomials of interpolation on an infinite interval.* (Abstract 47-9-377.)

45. R. L. Moore: *On continua with dendratomic subsets.* (Abstract 47-11-488.)

46. J. M. Dobbie: *A certain method in additive number theory.* Preliminary report. (Abstract 47-9-360.)

47. R. D. James: *On the sieve method of Viggo Brun.* (Abstract 47-9-361.)

48. A. T. Brauer: *On the least quadratic non-residue.* (Abstract 47-9-357.)

49. Albert Whiteman: *Sums connected with the partition function.* (Abstract 47-7-300.)

50. Fritz Herzog: *On an application of a theorem by Burnside.* (Abstract 47-9-341.)

51. Gordon Pall: *An elementary alternative to Dirichlet's theorem.* (Abstract 47-11-465.)

52. G. S. Young: *On topologically flat subsets of certain spaces of R. L. Moore.* (Abstract 47-9-452.)

53. O. G. Harrold: *A mapping characterization of Peano spaces.* (Abstract 47-9-444.)

54. R. H. Fox: *Homotopy type and deformation retraction.* (Abstract 47-9-442.)

55. N. E. Steenrod: *Topological methods for the construction of tensor functions.* (Abstract 47-11-489.)

56. Max Dehn: *On the mapping of closed surfaces.* (Abstract 47-9-440.)

57. A. W. Tucker: *A boundary-value theorem for harmonic tensors.* (Abstract 47-9-449.)

58. Samuel Eilenberg and Saunders MacLane: *Infinite cycles and homologies.* (Abstract 47-11-487.)

59. Henry Wallman: *Dimension for general spaces.* (Abstract 47-11-490.)

60. R. P. Agnew and Mark Kac: *On differences of elements of sets having positive Haar measure.* (Abstract 47-9-365.)

61. P. R. Halmos and John von Neumann: *Operator methods in classical mechanics.* II. (Abstract 47-9-385.)

62. Shizuo Kakutani and Warren Ambrose: *Structure and continuity of measurable flows* (Abstract 47-9-389.)

63. Szolem Mandelbrojt: *New proof of the conditions of quasi-analyticity.* (Abstract 47-9-393.)

64. J. W. T. Youngs: *A generalized Lebesgue integral.* (Abstract 47-9-409.)

65. Arthur Rosenthal: *On differentiation of integrals and approximate continuity.* (Abstract 47-9-401.)

66. R. B. Kershner: *On the packing of convex regions in the plane.* Preliminary report. (Abstract 47-9-390.)

67. A. D. Campbell: *On the application of barycentric coordinates to lattice theory.* (Abstract 47-9-338.)

68. A. A. Albert: *Non-associative algebras.* I. (Abstract 47-9-331.)

69. L. R. Wilcox: *Complementation and modularity in lattices.* (Abstract 47-9-356.)

70. Richard Brauer: *On the indecomposable representations of algebras.* (Abstract 47-9-334.)

71. R. M. Thrall: *On symmetrized Kronecker powers and Lie representations of the full linear group.* (Abstract 47-9-353.)

72. G. K. Kalisch: *On Jordan algebras.* (Abstract 47-9-344.)

73. Wilfred Kaplan: *Topology of the two-body problem.* (Abstract 47-9-412.)

74. Eric Reissner: *On the equations of the stability theory of thin shells.* (Abstract 47-9-415.)

75. Isaac Opatowski: *On the theory of lethal irradiation of micro-örganisms.* I. (Abstract 47-9-414.)

76. A. S. Householder: *A theory of steady-state activity in nerve-fiber networks.* IV: *n circuits with a common synapse.* (Abstract 47-9-411.)

77. W. K. Feller: *On the integral equation of the renewal theory.* (Abstract 47-9-435-*t.*)

78. Hilda P. Geiringer: *Some observations on analysis of variance theory.* (Abstract 47–11-484-*t.*)

79. Raphaël Salem: *On sets of multiplicity for trigonometrical series.* (Abstract 47-9-402-*t.*)

80. T. F. Glass and Walter Leighton: *On the convergence of continued fractions.* (Abstract 47-9-384-*t.*)

81. J. P. LaSalle: *A characterization of topological spaces.* (Abstract 47-9-447-*t.*)

82. A. D. Wallace: *Regularly ordered systems of sets.* (Abstract 47-9-450-*t.*)

83. Ralph Mansfield: *Differential systems involving k-point boundary conditions.* (Abstract 47-9-394-*t.*)

84. H. A. Simmons: *Classes of maximum numbers associated with two symmetric equations in n reciprocals.* (Abstract 47-9-351-*t.*)

85. Stefan Bergman and D. C. Spencer: *Some properties of pseudo-conformal transformations in the neighborhood of boundary points.* (Abstract 47-9-368-*t.*)

86. B. W. Jones: *On an extension of a theorem of Witt.* (Abstract 47-9-343-*t.*)

87. Alexander Weinstein: *On the buckling of a rectangular clamped plate compressed in one direction.* (Abstract 47-9-416-*t.*)

88. Edward Kasner and J. J. DeCicco: *Geometry of dual-velocity systems.* (Abstract 47-5-265-*t.*)

89. Edward Kasner and J. J. DeCicco: *General differential geometry of second order differential elements.* (Abstract 47-7-321-*t.*)

90. Edward Kasner and J. J. DeCicco: *Conformal geometry of series of third order differential elements.* (Abstract 47-7-320-*t.*)

91. Edward Kasner and J. J. DeCicco: *Pseudo-conformal geometry: functions of two complex variables.* (Abstract 47-9-424-*t.*)

92. J. J. DeCicco: *Equilong geometry of series of collineal third order differential elements.* (Abstract 47-7-319-*t.*)

93. E. N. Oberg: *Notes on the approximation of a function by sums of orthonormal functions.* Preliminary report. (Abstract 47-7-311-*t.*)

94. T. A. Botts: *On convex sets in linear normed spaces.* (Abstract 47-7-327-*t.*)

95. J. D. Mancill: *On the solutions of a certain class of partial differential equations.* (Abstract 47-7-310-*t.*)

96. A. W. Jones: *On the characterization of groups whose lattices satisfy specified lattice identities.* (Abstract 47-7-295-*t.*)

97. C. J. Everett: *Vector spaces over rings.* (Abstract 47-7-294-*t.*)

98. Nathaniel Coburn: *Semi-analytic unitary subspaces of unitary spaces.* (Abstract 47-9-419-*t.*)

99. Gordon Pall: *Quaternions and sums of three squares.* (Abstract 47-9-350-*t.*)

100. P. W. Carruth: *Arithmetic of ordinals with applications to the theory of ordered abelian groups.* (Abstract 47-9-339-*t.*)

101. Marston Morse and G. A. Hedlund: *Manifolds without conjugate points.* (Abstract 47-9-425-*t.*)

102. E. A. Knobelauch: *Extension of homeomorphisms on the torus and projective plane.* (Abstract 47-9-445-*t*.)

103. J. D. Mancill: *Multiple integral problems of the calculus of variations with prescribed transversality coefficients.* (Abstract 47-9-392-*t*.)

104. Louis Weisner: *Roots of certain classes of polynomials.* (Abstract 47-9-355-*t*.)

105. František Wolf: *On the summability of trigonometrical integrals.* (Abstract 47-9-408-*t*.)

106. Orrin Frink: *Topology in lattices.* (Abstract 47-9-443-*t*.)

107. M. M. Day: *Ergodic theorems for abelian semi-groups.* (Abstract 47-9-372-*t*.)

108. M. M. Day: *Operations in Banach spaces.* (Abstract 47-9-373-*t*.)

109. H. L. Garabedian: *Theorems relating to the Cesàro kernel transformation.* (Abstract 47-9-382-*t*.)

110. H. L. Garabedian: *A class of linear integral transformations.* (Abstract 47-9-381-*t*.)

111. Deane Montgomery and Leo Zippin: *A theorem on Lie groups.* (Abstract 47-9-346-*t*.)

112. Garrett Birkhoff: *Metric foundations of geometry.* (Abstract 47-9-428-*t*.)

113. Olga Taussky and Ernest Best: *A class of groups.* (Abstract 47-9-352-*t*.)

114. A. A. Albert: *Division algebras over a function field.* (Abstract 47-9-330-*t*.)

115. D. C. Spencer: *A function-theoretic identity.* (Abstract 47-9-403-*t*.)

116. C. C. Camp: *A convergence proof involving an inseparable multiple contour integral.* (Abstract 47-9-371-*t*.)

117. S. A. Jennings: *Central chains of ideals in an associative ring.* (Abstract 47-9-342-*t*.)

118. R. P. Agnew: *On Hurwitz-Silverman-Hausdorff methods of summability.* (Abstract 47-9-364-*t*.)

119. R. H. Bruck and T. L. Wade: *On a tensor algebra of Cramlet.* (Abstract 47-9-336-*t*.)

120. T. L. Wade: *Euclidean concomitants of the ternary cubic.* (Abstract 47-9-354-*t*.)

121. R. H. Bruck: *The number of absolute invariants of a tensor.* (Abstract 47-9-335-*t*.)

122. Mark Kac: *Convergence and divergence of non-harmonic gap series.* (Abstract 47-9-387-*t*.)

123. Shizuo Kakutani: *A class of examples of mixing flows.* (Abstract 47-9-388-*t.*)

124. E. L. Buell: *On the distribution of plane stress in a semi-infinite plate with partially stiffened edge.* Preliminary report. (Abstract 47-9-410-*t.*)

125. G. S. Young: *On the outer boundaries of certain connected domains.* (Abstract 47-9-451-*t.*)

126. Abe Gelbart: *On some properties of mapping functions.* (Abstract 47-9-383-*t.*)

127. M. G. Moore: *On functions satisfying two functional equations.* (Abstract 47-9-396-*t.*)

128. Richard Brauer: *Investigations on group characters.* (Abstract 47-9-333-*t.*)

129. Brockway McMillan: *Absolutely monotone functions.* Preliminary report. (Abstract 47-11-471-*t.*)

130. Ivan Niven: *Quadratic diophantine equations in the rational and quadratic fields.* (Abstract 47-9-362-*t.*)

131. Ivan Niven: *The mth roots of a quaternion.* (Abstract 47-9-348-*t.*)

132. L. R. Wilcox: *Extensions of semi-modular lattices.* II. (Abstract 47-11-461-*t.*)

133. Leonard Carlitz: *Note on generalized Bernoulli and Euler numbers.* (Abstract 47-9-358-*t.*)

134. Leonard Carlitz: *The coefficients of the reciprocal of a series.* (Abstract 47-9-359-*t.*)

135. E. H. Nicholson: *On the degree of approximation in some convergence theorems concerning derivatives of the mapping function in conformal mapping.* (Abstract 47-9-397-*t.*)

136. S. M. Ulam and D. H. Hyers: *On approximate isometries.* Preliminary report. (Abstract 47-9-427-*t.*)

137. S. M. Ulam: *On measures for subsets of sets of measure zero.* (Abstract 47-9-405-*t.*)

138. E. F. Beckenbach: *Vector formulations of Morera's theorem.* (Abstract 47-9-367-*t.*)

139. Paul Erdös and Gabor Szegö: *On a problem of I. Schur.* (Abstract 47-9-378-*t.*)

140. Reinhold Baer: *Automorphism rings of primary abelian operator groups.* (Abstract 47-9-332-*t.*)

141. H. S. Vandiver: *An arithmetical theory of Bernoulli numbers.* (Abstract 47-11-466-*t.*)

W. L. AYRES,
Associate Secretary

BOOK REVIEWS

Mathematics Applied to Electrical Engineering. By A. G. Warren. New York, Van Nostrand, 1940. 384 pp., 131 figs. $4.50.

This is a review course in college mathematics for electrical engineers. Some pages read as if they were a part of a mathematical text in electricity and magnetism, except that the emphasis is on mathematics; other pages read as if they were out of a book on "unified" college mathematics; in its entirety the book is a compact presentation of a number of useful mathematical principles and methods, illustrated by a wide selection of electrical problems.

Thus we find: complex numbers and their use in steady state electric circuit problems; linear differential equations with constant coefficients and their application to transients; the Bessel equation and the skin effect problem; partial differential equations and a discussion of plane waves, spherical waves and waves along wires; elements of functions of complex variable and problems of steady current flow; Heaviside's operational calculus and transients in transmission lines.

The above is not a complete list of either the kinds of mathematics or types of problems (some more elementary than others), which are taken up in the book. To help those who may be "rusty" on the principles of electricity and magnetism the author provides brief review chapters. Altogether, the book may appeal to college students and to practicing engineers—to the latter as a review book in fundamentals and to the former as a bridge between their mathematical and engineering studies.

SERGEI A. SCHELKUNOFF

Elemente der Operatorenrechnung mit geophysikalischen Anwendungen. By Hans Ertel. Berlin, Springer, 1940. 6+133 pp. RM 7.50.

This monograph demonstrates the applications of the operational calculus according to Heaviside to problems in geophysics. In the first section some pertinent remarks are made about certain simple differential equations with the view of later applications. The second section is concerned with the derivation and a brief physical discussion of some of the partial differential equations of mathematical physics, in particular the Stokes-Navier equations, the Maxwell equations and other equations derived from these. The third and fourth sections are concerned with a discussion of the Heaviside operational calculus and its applications to a variety of interesting problems in geophysics.

This text gives a brief but excellent account of the methods of mathematical physics. Despite the fact that this monograph is concerned primarily with the applications of the operational calculus to geophysics, one interested in a bird's-eye view of certain classical phases of mathematical physics can find them put down in short order. Many unusual problems of theoretical physics with geophysical applications are to be found.

A. E. HEINS

Matrix and Tensor Algebra for Engineers and Chemists. By Clarence E. Rose. Brooklyn, Chemical Publishing Co., 1940. 8+143 pp. $4.00.

Matrix and Tensor Algebra has been written "for electrical and communication engineers and chemists who wish to learn about determinants, vectors and tensors." Its only mathematical interest lies in the complete novelty of its point of view.

For example, no need is felt for proofs; theorems are stated as *obiter dicta*. Such definitions as "a matrix is a group of independent but related quantities," "a determinant is a finite, imaginary, or complex quantity, or generalization of the algebraic number zero" illustrate its unorthodoxy further. However, as the book was not designed for mathematicians, it would be out of place to go into more detail.

GARRETT BIRKHOFF

Compendio di Meccanica Razionale. 2d revised edition. By Tullio Levi-Civita and Ugo Amaldi. Bologna, Zanichelli, 1938. 12+423 +8+310 pp.

The two thousand page work by the same authors, entitled *Lezioni di Meccanica Razionale*, has already been adequately reviewed by K. P. Williams (this Bulletin, vol. 36, p. 781). It is one of the great standard works on mechanics of almost an encyclopaedic nature.

The Compendio, the work now under review, represents an attempt to cut down the Lezioni to the usual academic course of study in Italian universities and technical schools. This has been done by wholesale omissions of entire subjects rather than by abridged exposition. On the contrary, the emphasis on the foundations of the subject has been strengthened by a certain amount of additional material. Also included are a few minor corrections and revisions. For the most part, however, whole sections of the Compendio read word for word exactly as do the corresponding sections in the Lezioni.

Some of the subjects omitted in the Compendio are rather sur-

prising. We find, for example, no mention of the principle of least action, of Hamilton's canonical equations, or a fortiori of the Hamilton-Jacobi partial differential equation, or of invariant integrals. Thus there is necessarily no account of much of the important progress made during the last fifty years on the nature of conservative holonomic dynamical systems.

The length of the Compendio is further reduced by the omission of all exercises; for these the authors recommend the use of a book by Bisconcini (*Esercizi e Complementi di Meccanica Razionale*).

With all these omissions the work still runs to over seven hundred pages and thus still partakes something of an encyclopaedic nature in the more elementary parts of the subject. The balance between the statics and kinematics of the first part and the dynamics of the second part is, in our opinion, unfortunate. We should prefer a smaller first part and a larger second, especially inasmuch as the second part also contains a small amount of hydrodynamics and elasticity theory. For this reason we doubt the feasibility of using this work as a textbook in an American university, even if it were translated into English. But as a clearly and charmingly written reference book it might well prove invaluable.

D. C. LEWIS

Barlow's Tables of Squares, Cubes, Square Roots, Cube Roots and Reciprocals of all Integer Numbers up to 12,500. Edited by L. J. Comrie. 4th edition. London, Spon; Brooklyn, Chemical Publishing Co., 1941. 12+258 pp. $3.00.

The book is accurately described by its title. Roots are given to six places of decimals, and reciprocals to seven significant figures. The directions given for interpolation and for computing additional figures are adapted to the use of computing machines. There is appended a short table of powers of integers from 1 to 100 up to the tenth, and of those from 1 to 10 up to the twentieth.

Barlow's Tables are of historical interest. First prepared and published in 1814 by Peter Barlow of the Royal Military Academy (Woolwich), they were edited and reissued by A. De Morgan in 1839. The plates ultimately wore out and a third edition was prepared in 1930 by Dr. Comrie of H. M. Nautical Almanac Office. The present is a revision of the third edition, in which the extension from 10,000 to 12,500 is new. The prefaces to the first three editions are included in the present text.

R. A. JOHNSON

Introduction to Algebraic Theories. By A. Adrian Albert. The University of Chicago Press, 1941. 8+137 pp. $1.75.

Those who have learned abstract algebra know that its methods are at the same time simpler, easier to understand and more penetrating than the more classical procedures; but those who have taught abstract algebra know that there is one great obstacle in the way of the student desirous of acquainting himself with this new approach to algebra. The novice has first to learn to work with concepts, to "compute" with them as easily as he used to do with numbers and such like; though after having succeeded in this, he finds that everything else is comparatively easy going.

It is the object of this *Introduction to Algebraic Theories* to help the student in his attempt to overcome the difficulties just indicated and to acquire those habits of abstract thinking which are the indispensable foundation not only of abstract algebra, but of almost all present day mathematics. Since this book is intended to be a book of preparation, it does not require any previous knowledge—apart from algebraic techniques already known to the mathematicians of the Renaissance—though a willingness to work and think are indispensable.

The topics treated in this text are those which one would naturally expect: matrices, their equivalence and similarity, as well as a number of related subjects. This entails some study of polynomials—for technical reasons—and of linear spaces, since the latter are indispensable for understanding the real significance of the concepts pertaining to matrices. Indications of the technique of generalization—so important today—and of its usefulness are given throughout. The culmination of the whole work may be seen in the last chapter in which the fundamental concepts of abstract algebra are introduced as the natural outcome of the preceding considerations.

A great number of exercises—both numerical illustrations and mathematical applications—provide the reader with an opportunity to test the acquired skills. The book is written with the clarity of style, the arguments are presented with the elegance and precision which one has learned to expect from its author. Thus this text will prove valuable to student and teacher alike.

REINHOLD BAER

NOTES

Copies of the report *Mathematical Education for Defense* of the Subcommittee on Education for Service of the War Preparedness Committee (W. L. Hart, Chairman) may be obtained, while the supply lasts, in lots of 25 for $1.25 from Professor J. R. Kline, University of Pennsylvania, Philadelphia, Pa., or Professor W. D. Cairns, 97 Elm Street, Oberlin, Ohio. This report is reprinted from the American Mathematical Monthly and has also appeared in the Mathematics Teacher and School Science and Mathematics.

During the winter 1941–1942 a symposium on non-linear circuit theory will be presented by the Basic Science Group of the American Institute of Electrical Engineers, New York Section. The following six lectures will be given: *General nature of the problem of non-linear circuits and outline of the various methods of attack*, Professor E. Weber, Polytechnic Institute of Brooklyn; *Power series solutions*, Dr. L. A. MacColl, Bell Telephone Laboratories; *Fourier series solutions*, Mr. W. R. Bennett, Bell Telephone Laboratories; *Graphical methods*, Mr. A. Preisman, R.C.A. Institute; *Numerical step-by-step method*, Professor Alan Hazeltine, Stevens Institute of Technology; *Analytical methods*, Dr. P. Le Corbeiller, Harvard University and New York University. Further information about this symposium may be obtained from Mr. H. E. Farrer, A.I.E.E. Headquarters, 33 West 39th Street, New York, N. Y.

Applications for Benjamin Peirce instructorships at Harvard University for the year 1942–1943 should be sent to the Chairman of the Department of Mathematics. Candidates should have received the Ph.D. degree or have had equivalent training.

The School of Mathematics of the Institute for Advanced Study each year allocates a small number of stipends to gifted young mathematicians and mathematical physicists to enable them to study and to do research work at Princeton. Candidates must have given evidence of ability in research comparable at least with that expected for the degree of Doctor of Philosophy. Blanks for application may be obtained from the School of Mathematics of the Institute, Fuld Hall, Princeton, N. J., and are returnable by February 1, 1942.

The first Charles Chree Medal and Prize of the English Physical Society has been awarded to Professor Sydney Chapman of the Imperial College of Science and Technology.

According to an announcement in Mathematische Annalen (vol.

117, no. 5, p. 776) Jablonowskischen Gesellschaft der Wissenschaften is offering a prize of 500 RM for an investigation of older Greek arithmetic and algebra. Manuscripts must be in German or Latin and will be received up to the end of 1942.

Professor G. D. Birkhoff of Harvard University has been elected an honorary member of the London Mathematical Society.

An honorary doctorate of science has been conferred by Washburn College upon Professor L. M. Graves of the University of Chicago.

The honorary degree of doctor of laws has been conferred upon Dean R. G. D. Richardson of Brown University by Lehigh University.

Guggenheim Fellowships for 1941–1942 have been awarded to Professor Carrillo Flores of the National University of Mexico, Assistant Professor Richard Brauer of the University of Toronto, Associate Professor Deane Montgomery of Smith College, and Dr. Jesse Douglas.

Professor David van Dantzig of the Institute of Technology, Jaffalaan, Delft, Netherlands, has retired.

The following additional members of the American Mathematical Society are in residence at the Institute for Advanced Study for all or part of this academic year: Dr. Valentin Bargmann, Dr. David Blackwell, Dr. A. T. Brauer, Professor Richard Brauer of the University of Toronto, Dr. Norman Davids, Professor J. L. Doob of the University of Illinois, Dr. Guido Fubini, Dr. Kurt Gödel, Dr. P. R. Halmos, Dr. M. H. Heins, Dr. G. P. Hochschild, Professor Shizuo Kakutani of Osaka Imperial University, Mr. G. K. Kalisch, Mr. G. W. Mackey, Dr. Dorothy Maharam, Professor Deane Montgomery of Smith College, Dr. L. J. Savage, and Dr. R. M. Thrall of the University of Michigan.

The following persons are also to be at the Institute for Advanced Study during a part or all of the current academic year: Dr. Felix Adler, Professor Subrahmanyan Chandrasekhar of Yerkes Observatory, Professor K. S. Cole of Columbia University College of Physicians and Surgeons, Dr. S. M. Dancoff, Professor Wolfgang Pauli of Eidgenossische Technische Hochschule, Zürich, Dr. Hans Samelson, Professor Mario Schenberg of University of Sao Paulo, Brazil, Professor C. L. Siegel formerly of the University of Göttingen, and Mr. A. H. Stone.

Dr. H. R. Branson of Dillard University has been appointed to an assistant professorship at Howard University.

Associate Professor I. W. Burr of Antioch College has been appointed to an assistant professorship at Purdue University.

Dr. W. B. Caton of Athens College, Athens, Alabama, has been appointed acting head of the mathematics department at Southwestern College, Winfield, Kansas.

Dr. F. G. Dressel of Duke University has been promoted to an assistant professorship.

Professor Emeritus W. F. Durand of Stanford University has been appointed a member of the National Advisory Committee on Aeronautics.

Dr. Samuel Eilenberg of the University of Michigan has been promoted to an assistant professorship.

Professor R. E. Gaines of the University of Richmond has retired as head of the department of mathematics.

Dr. A. S. Galbraith of the University of Rochester has been appointed to an assistant professorship at Colby College.

Assistant Professor F. C. Gentry of Louisiana Polytechnic Institute has been promoted to an associate professorship.

Dr. W. N. Hallett has accepted a position at Frostburg State Teachers College, Frostburg, Maryland.

Professor A. M. Harding of the University of Arkansas has recently been made president of the University.

Professor I. M. Hostetter of Howard College, Birmingham, Alabama, has been appointed to an assistant professorship at Oregon State College.

Dr. R. N. Johanson of Bradley Polytechnic Institute has been promoted to an assistant professorship.

Dr. P. M. Kendig of the Schuylkill Undergraduate Center, Pennsylvania State College, has been appointed to an assistant professorship of physics at Franklin and Marshall College.

President E. O. Lovett of Rice Institute, formerly professor of mathematics and astronomy at Princeton University, has retired at the age of seventy with the title president emeritus.

Assistant Professor E. W. Miller of the University of Michigan has been promoted to an associate professorship.

Dr. C. J. Nesbitt of the University of Michigan has been promoted to an assistant professorship.

Dr. L. F. Ollmann of the College of Wooster has been appointed to an associate professorship at Hofstra College.

Dr. E. D. Rainville of the University of Michigan has been promoted to an assistant professorship.

Assistant Professor G. de B. Robinson of the University of Toronto has an appointment from the National Research Council of Canada.

Dr. R. M. Robinson of the University of California has been promoted to an assistant professorship.

Dr. O. K. Sagen has been appointed chief statistician of the Department of Public Health at Springfield, Illinois.

Dr. W. J. Schart of Ohio State University has been appointed professor of mathematics at the Aviation Cadet Replacement Center, Maxwell Field, Alabama.

Dr. C. E. Sealander of the State University of Iowa has been appointed to an assistant professorship at the University of South Dakota.

Mr. J. W. Sheedy of Michigan State College has been promoted to an assistant professorship.

Associate Professor I. S. Sokolnikoff of the University of Wisconsin has been promoted to a professorship.

Dr. Anna A. Stafford of the University of Utah has been promoted to an assistant professorship.

Associate Professor E. C. Stopher of Ashland College, Ashland, Ohio, has accepted a position at Brockport State Normal School, Brockport, New York.

Mr. E. A. Sturley of Yale University has accepted a position at Berkshire School, Sheffield, Massachusetts.

Dr. G. B. Van Schaack of Michigan State College has been promoted to an assistant professorship.

Dr. W. F. Whitmore of the University of California has accepted a position at the Naval Ordnance Laboratory, Washington, D. C.

Assistant Professor Louise A. Wolf of the University of Wisconsin at Milwaukee has been appointed lecturer at the University of Wisconsin.

Associate Professor R. V. Churchill of the University of Michigan is on leave of absence for the first semester of 1941–1942 and is visiting lecturer at the University of Wisconsin.

Assistant Professor C. H. Fischer of Wayne University is this year visiting assistant professor of mathematics at the University of Michigan.

Assistant Professor A. S. McMaster of the University of Colorado is on leave during the academic year 1941–1942 and is studying at the Iowa State College.

The following appointments to instructorships are announced: University of Chicago: Dr. P. V. Reichelderfer; Colorado College: Dr. Margaret M. Hansman; Cornell University: Dr. Joseph Lehner, Mr. J. C. Smith; Illinois Institute of Technology: Dr. W. S. Snyder; Indiana Technical College: Mr. H. S. Kieval; University of Maryland: Dr. L. C. Hutchinson; Michigan State College: Dr. E. E. Blanche, Mr. Vladimir Morkovin; University of Michigan: Dr. C. J. Thorne; Mundelein College, Chicago, Illinois: Mr. E. L. Godfrey; New Mexico College of Agriculture and Mechanic Arts: Mr. J. A. Joseph; Northwestern University: Dr. S. J. Lawwill, Mr. J. F. Paydon; Ohio State University: Mr. H. D. Huskey, Mr. C. G. Maple; University of Oklahoma: Dr. Ingo Maddaus; Oregon State College: Dr. Rhoda Manning; Pennsylvania State College: Dr. Abraham Schwartz; University of Pennsylvania; Mr. E. K. Ritter; Phillips Academy, Andover, Massachusetts: Mr. W. B. Morgan; Princeton University: Dr. J. H. Giese, Dr. M. S. Macphail, Dr. Henry Scheffé; Purdue University: Dr. G. W. Whitehead, Dr. J. W. T. S. Youngs; University of Richmond: Mr. F. B. Key; Agricultural and Mechanical College of Texas: Mr. W. B. Coleman; University of Texas: Dr. J. P. LaSalle; United States Naval Academy: Mr. T. J. Benac, Dr. E. E. Betz; Wells College: Miss Mary D. Clement; State Teachers College, River Falls, Wisconsin: Mr. C. J. Kirchen; University of Wisconsin: Mr. H. N. Laden; College of Wooster: Mr. H. L. Meyer; Yale University: Dr. R. P. Dilworth.

Dr. Kazimierz Bartel, formerly professor of mathematics at the

University of Lwow and at the Polytechnic Institute, is reported to have been killed.

The death of Mr. John Brill, London, England, has been reported.

Professor Emeritus Alfred Pringsheim of the University of Munich died in Zurich on June 25, 1941 at the age of ninety-one years.

Professor Emeritus W. H. Butts of the University of Michigan died June 25, 1941.

Professor O. C. Edwards of the University of Minnesota died July 17, 1941.

Professor Emeritus J. W. Glover of the University of Michigan died July 15, 1941 at the age of seventy-two years. He had been a member of the Society since 1896.

The death of Mr. Pomeroy Ladue has been reported. He had been a member of the Society since 1894.

Professor C. E. Magnusson of the University of Washington died July 10, 1941. He had been a member of the Society since 1926.

Professor Emeritus Alice Winbigler of Monmouth College died May 27, 1941.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

454. R. H. Bruck and T. L. Wade: *The number of independent components of the tensor* ${}_{[\alpha]}T_{i_1\cdots i_p}$.

Let $T_{i_1\cdots i_p}=\sum_\alpha {}_{[\alpha]}T_{i_1\cdots i_p}$ represent the decomposition of an arbitrary tensor $T_{i_1\cdots i_p}$ into tensors of various symmetry types, a type corresponding to each partition $[\alpha]$ of the indices $i_1\cdots i_p$ (H. Weyl, *The Classical Groups*, pp. 96–136; T. L. Wade *Tensor algebra and Young's symmetry operators*, American Journal of Mathematics, vol. 63 (1941) pp. 645–657). J. A. Schouten (*Der Ricci-Kalkül*, chap. 7, §7) considers the problem of finding the number of independent scalar components of c_α of the tensor ${}_{[\alpha]}T_{i_1\cdots i_p}$ and obtains expressions for c_α in terms of the dimension n of the coordinate system in the cases $p=2, 3, 4$, but the difficulties of his method become great for $p\geqq 5$. In this note it is shown that c_α is equal to r_α, the rank, recently defined by the authors, of the immanent tensor ${}_{[\alpha]}T^{i_1\cdots i_p}_{j_1\cdots j_p}$ (abstract 47-5-185). The general formulas for the r_α's in terms of n may be obtained directly from the character table of the symmetric group on p letters whenever this is available, as for $p\leqq 13$. Tables of $c_\alpha=r_\alpha$ are given for $p\leqq 6$. (Received October 1, 1941.)

455. A. D. Campbell: *On the application of an algebra of sets to group theory.*

In this paper is assumed the existence of a set S of abstract entities (including 0 and 1) whose subsets are used as coordinates for groups, subgroups, elements of groups, sets of elements, and neighborhoods (in the study of topological groups). The relation $\alpha\subset\beta$ means that the element (or set or group or neighborhood) α is contained in the set (or group or neighborhood) β. By (α, β) is meant the join of α and β, by $\alpha\cdot\beta$ the meet of α and β. The unit element is called 1, the null-set (or null-group) is called 0. By $\alpha\beta$ is meant the set consisting of all products of elements of α by elements of β (in this order). Obvious meanings are given to $\alpha-\beta$ and α^{-1}. It is agreed that $[\alpha_1, \alpha_2, \cdots, \alpha_n]$ shall mean the group generated by $\alpha_1, \alpha_2, \cdots, \alpha_n$ and that $0\alpha=\alpha 0=0$, $1\alpha=\alpha 1=\alpha$, $\alpha\cdot 0=0$, $\alpha-0=\alpha$. It is to be noted that $(\alpha, \beta)\gamma=(\alpha\gamma, \beta\gamma)$ and $(\alpha-\beta)\gamma=\alpha\gamma-\beta\gamma$ and $(\alpha\cdot\beta)\cdot\gamma=(\alpha\gamma)\cdot(\beta\gamma)$; also that for a group $\alpha\alpha=\alpha$, $\alpha\alpha^{-1}=\alpha$, $\alpha^{-1}=\alpha$. By this algebra old theorems are readily proved and new ones are derived. Thus it follows that if $\alpha=(\beta, \gamma)$ is a finite group of order a with β as a subgroup of order b and with $\beta\cdot\gamma=0$, then the relation $\beta\alpha=(\beta\beta, \beta\gamma)=(\beta, \beta\gamma)$ shows clearly (since $\beta\cdot(\beta\gamma)=0$) the well known result that $a=b+bc=b(1+c)$. (Received September 15, 1941.)

456. Tomlinson Fort: *Generalizations of the Bernoulli polynomials and numbers and corresponding summation formulas.*

The operators d/dx and $\sum$ can be used to define the Bernoulli polynomials. These are generalized to linear operators by means of which very general classes of polynomials are defined, the coefficients of which serve to generalize the Bernoulli numbers. The polynomials determined are very general and include as special cases not only the classical Bernoulli polynomials, but the Bernoulli polynomials of the second kind, the Bernoulli and Euler polynomials of higher order as defined by Nörlund (*Differenzenrechnung*, p. 119) and the Bernoulli numbers as generalized by Vandiver (Proceedings of the National Academy of Sciences, vol. 23, p. 555). Attention is called to several new and interesting classes of polynomials and corresponding numbers. The polynomials of the paper are made the starting point for summation formulas analogous to Taylor's formula and to the Euler-Maclaurin formula of classical mathematics. A form for the remainder is obtained analogous to the Lagrange form for the remainder in Taylor's formula. (Received September 18, 1941.)

457. D. H. Lehmer: *Properties of the coefficients of the modular invariant $J(\tau)$.*

The properties of the integer coefficients in the Fourier series development of Klein's absolute elliptic modular invariant $j(\tau)=12^3J(\tau)=e^{-2\pi i\tau}+744+196884e^{2\pi i\tau}+\cdots=\sum_{n=-1}^{\infty}c(n)\exp(2\pi in\tau)$ are of two sorts: (a) congruence properties with respect to small moduli and (b) multiplicative properties. The congruence properties are similar to those of Ramanujan's numerical function $\tau(n)$ with which the c's are intimately related. For example, $(k+1)c(k)\equiv 0 \pmod{24}$. Multiplicative properties are discovered which enable one to express $c(kn)$ simply in terms of previous c's for $n<k+1$. For example, $c(10)=c(6)+c(1)c(4)+c(2)c(3)$. Similar properties hold for the coefficients of any positive integral power of $j(\tau)$. (Received September 30, 1941.)

458. Howard Levi: *A characterization of polynomial rings by means of order relations.*

In this paper rings are considered for whose elements certain order relations and algorithms have been assumed. The assumptions made were borrowed from polynomial rings, where they appear as fairly obvious facts. It is shown that these facts characterize polynomial rings; that any ring for which they are postulated must be a polynomial ring. In view of the fact that polynomial rings with different types of coefficient domains differ significantly, two lists of assumptions are presented. The first leads to a ring of polynomials whose cofficients constitute a domain of integrity which contains a unit element and in which the Hilbert basis theorem holds. The coefficient domain derived with the second list is a field. These two lists of assumptions differ only slightly, and minor modifications of them could be made which would lead to more general types of coefficients. These cases are not discussed, in the belief that a polynomial whose cofficients are of a very general type has few claims to attention which it does not share with any element of a general ring. (Received September 25, 1941.)

459. R. J. Levit: *Fields in terms of a single operation.*

Though it is customary to consider fields as based on two independent operations, they may equally well be regarded as systems of single composition. This was first

shown by N. Wiener when he exhibited a set of seven postulates for fields in terms of a single operation. In the present paper are presented two other definitions of fields each in terms of a single operation and requiring five and six postulates respectively. The former takes as primitive an operation expressible as $a(1-b)$, which has the advantage of being class-closing. The latter definition uses Wiener's operation interpreted as $1-a/b$, which, though not class-closing, makes it possible to dispense with all existence postulates outside of the closure conditions. Detailed proofs are supplied of the necessity, sufficiency, consistency, and independence of both sets. A number of other single operations that can be used to define fields are also discussed. It is shown that no field-defining operation (or set of operations) is uniquely characterized by the postulates it satisfies. Each such operation (or set) is susceptible of a variety of interpretations, and all such interpretations are found. (Received October 1, 1941.)

460. Ernst Snapper: *Co-maximal linear sets and products of linear sets.*

The linear subsets of an n-dimensional vector space V_n satisfy the Noether decomposition theory (this Bulletin, abstract 47-3-124). The following three definitions are introduced: The product of two linear sets L_1 and L_2 is $(\mathfrak{E}_1L_2+\mathfrak{E}_2L_1)$, where $\mathfrak{E}_i$ is the essential ideal of L_i. Two linear sets are co-maximal if their sum is the whole vector space. Two linear sets are strongly co-maximal if the sum of their essential ideals is the whole scalar domain. Strongly co-maximal linear sets are co-maximal. The definitions become the familiar definitions of co-maximal ideals and products of ideals if $n=1$. The following theorems are proved: The intersection of a finite number of linear sets, strongly co-maximal in pairs, equals their product. Every linear set is the irredundant intersection of a finite number of linear sets, strongly co-maximal in pairs; the intersection components themselves are not irredundant intersections of strongly co-maximal pairs and are unique. The results are used to study the decomposition of the restclass group V_n/L into a direct sum of subgroups. (Received September 8, 1941.)

461. L. R. Wilcox: *Extensions of semi-modular lattices.* II.

This paper generalizes a theorem of the author concerning complemented semi-modular lattices L (abstract 47-5-208) to the case where L is of dimension greater than or equal to 4 (that is, there exists in L a chain $a_1>a_2>\cdots>a_n$, where $n\geqq 6$), and L possesses no two parallel hyperplanes. (Received August 7, 1941.)

462. R. E. O'Connor: *A theorem of Diophantine approximation and an application to the values of a linear form.*

Assume θ given, $\eta\geqq 1$, x, y rational integers and all other numbers real. Let the proposition P_1 assert that $c=c(\theta)$ exists such that, for every $t>1$ and every α, the non-homogeneous system, $0<x<ct^{\eta}$, $|x\theta-y-\alpha|<t^{-1}$, has a solution (x, y). Let P_2 assert that a positive γ exists such that, for every $t>1$, the homogeneous system, $0<x<\gamma t^{1/\eta}$, $|x\theta-y|<t^{-1}$, is insoluble. Referring to the standard classification of irrationals (see Koksma, *Diophantische Approximationen*, p. 27), it is easily shown that *P_2 is equivalent to the proposition that θ is irrational and of type lower than $I\eta\infty$*. In the present paper it is established that *P_1 and P_2 are equivalent propositions*, a theorem proved in 1924 by A. Khintchine for the case $\eta=1$. Now let ω, ω' be positive numbers

with ratio θ and let $\mu_0=0, \mu_1, \mu_2, \cdots$ be the sequence S_μ of values, in order of increasing magnitude, taken by the form $x\omega+y\omega'$ as x, y range through the non-negative integers. For every positive μ, define $\Delta(\mu)=\max_{\mu_{i+1}>\mu}(\mu_{i+1}-\mu_i)$. Employing the above theorems it is proved that $\Delta(\mu)=O(\mu^{-1/\eta})$ $(\mu\to\infty)$, *if and only if* θ *is irrational and of type lower than* $I\eta\infty$. Further information concerning the sequence S_μ is also given and extensions of the last theorem to cases with θ an irrational of higher type. (Received August 21, 1941.)

463. R. E. O'Connor: *Representation of integers by power-products of two integers.*

With a, b integers, $1<a<b$, let $A(n)$ be the number of representations of the positive integer n in the form $n=\sum c_i a^i b^i$, the coefficients being non-negative integers less than a. Then $A(n)$ *is equal to* $P(n)=P(b, n)$, the number of partitions of n into non-negative powers of b. By a modification of a method of Hardy and Ramanujan (Proceedings of the London Mathematical Society, (2), vol. 17 (1918), pp. 75–115) it is shown that $\log P(n)\sim(\log n)^2/2\log b$, *as* $n\to\infty$. The same result is established by showing, with $f(x)=1+\sum_{n=1}^{\infty}P(n)x^n(|x|<1)$, that $\log f(x)\sim(\log^2(1-x))/2\log b$ $(x\to1-0)$ and by using this asymptotic value of $\log f(x)$ in the following theorem of Tauberian type. *Assume*: (1) $\phi(t)=a_0+a_1e^{-t}+a_2e^{-2t}+\cdots$, $a_i\geqq0$ $(i=0, 1, 2, \cdots)$, *converges for* $t>0$; (2) $\log\phi(t)=(1+o(1))K(\log(t^{-1}))^\alpha$ $(t\to0)$, $K>0$, $\alpha>0$; (3) $A_n=\sum_{i=0}^{n}a_i$ $(n=0, 1, 2, \cdots)$. Then $\log A_n=(1+o(1))K(\log n)^\alpha$ $(n\to\infty)$. (Received August 21, 1941.)

464. R. E. O'Connor: *Representation of integers by power-products of two real numbers.*

Let α, β *be real numbers greater than unity with* $\theta=\log\alpha/\log\beta$ *irrational.* Define $M(n)$ as the greatest integer such that every integer M', $0<M'\leqq M$, has a representation of the form $M'=[\alpha^{r_1}\beta^{s_1}]+[\alpha^{r_2}\beta^{s_2}]+\cdots+[\alpha^{r_\nu}\beta^{s_\nu}]$ where the r_i, s_i are non-negative integers and $\nu\leqq n$. Very simple considerations show that *to any* $\epsilon>0$ *there corresponds an* $n_0=n_0(\epsilon, \alpha, \beta)$ *such that* $\log M(n)<(2+\epsilon)n\log n$, *for* $n>n_0$. Lower bounds for $M(n)$ are obtained in two cases as follows. *Assume* θ *is of type lower than* $I\eta\infty$; *then for every positive* k *there is an* $n_0=n_0(k, \alpha, \beta)$ *such that* $\log M(n)>\eta^{-1}n\log n+kn$ $(n>n_0)$. Or, *assume* α, β *algebraic; then for every positive* ϵ *there is an* $n_0=n_0(\epsilon, \alpha, \beta)$ *such that* $\log M(n)>n(\log n)^{1/2-\epsilon}$ $(n>n_0)$. In establishing the first of these lower bounds use is made of the theorems of abstract 47–11–462; for the second is required a theorem of Gelfond (Bulletin de l'Académie des Sciences de l'URSS, Série Mathématique, 1939, pp. 509–518). (Received August 21, 1941.)

465. Gordon Pall: *An elementary alternative to Dirichlet's theorem.*

It is shown how the application of Dirichlet's theorem on primes in an arithmetical progression can be replaced in many problems (especially concerning quadratic forms) by the application of Gauss's theorem on the existence of binary quadratic forms with assigned values for their generic invariants. For example, Meyer's theorem on indefinite quinary forms representing zero and the theorem giving conditions for quaternary zero forms can be proved by entirely elementary methods. (Received August 7, 1941.)

466. H. S. Vandiver: *An arithmetical theory of Bernoulli numbers.*

In this paper a method is described which enables one to find many new types of

arithmetical relations concerning the Bernoulli and allied numbers. It depends mainly on the following idea: Let a and b be rational, with $a \equiv b \pmod{p}$, where p is any prime integer. If a and b are independent of p it then follows, since there is an infinity of primes, that $a=b$. This is applied in connection with wha is perhaps the simplest formula in which a single Bernoulli number appears: $S_n^{(p)} \equiv pb_n \pmod{p^2}$, where $S_n(p)=1^n+2^n+\cdots+(p-1)^n$. The paper starts with simple identities involving $(x^p-1)/(x-1)$, obtains congruences modulo p or p^2 by differentiation and integration, and then makes substitutions for the variables which give congruences involving the Bernoulli numbers. Generalizations of the Staudt-Clausen theorem as well as analogues of the same are obtained. Proofs are given of nearly all the results which were stated without proof in two previous papers by the writer (Proceedings of the National Academy of Sciences, vol. 23 (1937), pp. 555–559; vol. 25 (1939), pp. 197–201). This article will appear in the Transactions of this Society. (Received August 12, 1941.)

Analysis

467. Lipman Bers: *A convergence theorem for analytic functions of two variables.*

Let M be a domain of the four-dimensional z_1, z_2-space ($z_k=x_k+iy_k, k=1,2$) bounded by the two analytic hypersurfaces $E[z_1 \epsilon B(z_2)+c(z_2), z_2=e^{i\theta}, 0 \leqq \theta < 2\pi]$ and $E[z_1 \epsilon c(z_2), |z_2|<1]$ where $B(z_2)$ is a star domain bounded by the curve $c(z_2)=E(z_1=h(z_2, \lambda), 0 \leqq \lambda < 2\pi]$ and $h(z_2, \lambda)$ is an analytic function of z_2, $|z_2| \leqq 1$, for every fixed value of λ, and is subject to certain additional conditions. Consider a sequence of analytic functions $\{f_n(z_1, z_2)\}$ defined in M and satisfying the condition $\int_0^{2\pi}\int_0^{2\pi} |f[r_1 h(r_2 e^{i\theta}, \lambda), r_2 e^{i\theta}]|^p d\theta d\lambda < K$, $0<r_k<1$, $k=1, 2$, $n=1, 2, \cdots$, where p is a fixed number greater than 1. It is known that $f_n(z_1, z_2)$ possesses finite sectorial limits $(F_n(\theta, \lambda)$ almost everywhere on the intersection $F=E[z_1=h(e^{i\theta}, \lambda), z_2=e^{i\theta}, 0 \leqq \theta < 2\pi, 0 \leqq \lambda \leqq 2\pi]$ of the two boundary hypersurfaces of M (see Bergman and Marcinkiewicz, Fundamenta Mathematicae, vol. 33 (1939), pp. 75–94, and a paper by the author to appear in the American Journal of Mathematics). If the sequence $\{F_n(\theta, \lambda)\}$ converges in a set of positive two-dimensional measure, the sequence $\{f_n(z_1, z_2)\}$ converges uniformly in every closed subdomain of M. This theorem can be generalized, in a modified form, for more general types of domains. (Received September 27, 1941.)

468. A. B. Brown: *Independent parameters for sets of functions.*

The results previously announced for the case of one function of n variables and m parameters (under the title *On the number of independent parameters*, abstract 46-11-485) are now extended to the case of a set of r functions of n variables and m parameters. The methods and results are similar to those for the case of one function. An additional theorem is given, on invariance of the number of parameters in a "complete" set under certain transformations of parameters. Only one paper is offered for publication, covering the more general case announced here, but under the original title. (Received September 11, 1941.)

469. E. R. Lorch: *The theory of analytic functions in normed abelian vector rings.* Preliminary report.

A complex vector space $\mathfrak{R}$ for which the commutative multiplication of elements is defined is here called a normed abelian vector ring if the norm satisfies

$|A \cdot B| \leqq |A| \cdot |B|$. $\mathfrak{R}$ is assumed to possess a unit I for which $|I| = 1$. For those rings in which the singular elements (those without inverse) are precisely the frontier of the group of nonsingular elements, the classic theory of analytic functions is developed. Analytic functions are defined to be those mapping domains in $\mathfrak{R}$ upon $\mathfrak{R}$ for which the derivative exists. The theory elaborated for these functions includes the Cauchy theorem and formula and the power series representation. In addition, various known results on normed rings are derived anew by the author's methods. (Received September 3, 1941.)

470. A. N. Lowan and William Horenstein: *On the function* $H(m, a, x) = \exp(-ix)F(m+1-ia, 2m+2, 2ix)$.

The substitutions $\alpha = m+1-ia$, $\gamma = 2m+2$ and $u = 2ix$, in a known integral representation of the confluent hypergeometric function $F(\alpha, \gamma, x)$ led to the following interesting results: (1) A recurrence formula between the values of $H(m, a, x)$ for three consecutive integral values of the parameter m. (2) A recurrence formula between the values of $H(m, a, x)$ and $(\partial/\partial x)H(m, a, x)$ for two consecutive integral values of parameter m. (3) A recurrence formula between $H(m, a, x)$ and $(\partial/\partial x)$ $H(m, a, x)$ for the integral value of the parameter m, and the value of $H(m+1, a, x)$. (4) A recurrence formula between the value of $H(m, a, x)$ for an integral value of the parameter m and the values of $H(m+1, a, x)$ and $(\partial/\partial x)$ $H(m+1, a, x)$. The above results were obtained in the course of some exploratory study of the function $H(m, a, x)$ now being carried out by the Mathematical Tables Project conducted by the Work Projects Administration for the City of New York under the sponsorship of the National Bureau of Standards. (Received August 5, 1941.)

471. Brockway McMillan: *Absolutely monotone functions.* Preliminary report.

Let $\mathcal{A}$ be an additive complemented family of subsets of some space S. Let $\mathcal{A}^N$ be the additive complemented family determined by the Cartesian product sets $a_1 \otimes \cdots \otimes a_N$. Let $\mathcal{P}$ be the class of finite partitions π of S by pairwise disjoint members of $\mathcal{A}$, $\mathcal{P}$ being ordered by refinement. If $F(x)$ is defined for $x \in \mathcal{A}$, let $\Delta^0 F(x) = F(x)$ and $\Delta^N F(x;\ a_1 \otimes \cdots \otimes a_N) = \Delta^{N-1} F(x+a_N;\ a_1 \otimes \cdots \otimes a_{N-1})$ $-\Delta^{N-1} F(x;\ a_1 \otimes \cdots \otimes a_{N-1})$. If these differences are non-negative for every N and every collection of pairwise disjoint $a_i \in \mathcal{A}$ and if $u \in \mathcal{A}^N$, the limit on $\mathcal{P}$ $F^N(u) = \lim \{\sum \Delta^N F(0;\ a_1 \otimes \cdots \otimes a_N \cdot u) \mid \pi \in \mathcal{P}\}$ exists for every N, the summation being taken over all combinations $a_1 \otimes \cdots \otimes a_N$ of distinct elements of π. If $G(x) = F^N(x^N)$, then $\Delta^{N+1} G(x;\ a_1 \otimes \cdots \otimes a_{N+1}) = 0$ identically in x and pairwise disjoint a_i. Furthermore, (i) $F(x) \geqq F(0) + \sum (1/n!) F^N(x^N)$. If $\Delta^{K+1} F = 0$ identically, (i) is an equality and the right member terminates with the term $F^K(x^K)$. If (i) is an equality for some $x = x_0$, then equality holds for all $x \leqq x_0$. There exist families $\mathcal{A}$ and functions F for which (i) is not identically an equality. (Received August 14, 1941.)

472. G. W. Mackey: *On the lattice of closed linear subspaces of a normed linear space.*

Let X_1 and X_2 be normed linear spaces. Let L_1 and L_2 be the lattices of closed linear subspaces of X_1 and X_2 respectively. If L_1 and L_2 are isomorphic as lattices then, X_1 and X_2 are isomorphic as normed linear spaces, i.e., there exists a 1-1 linear transformation of all of X_1 into all of X_2 which is an homeomorphism. Eidelheit

(Studia Mathematica vol. 9 (1940) pp. 97–105) has proved an analogous theorem in which the ring of continuous linear transformations of the space into itself takes the place of the lattice of closed linear subspaces. He assumes in addition that X_1 and X_2 are complete. A strengthened form of Eidelheit's theorem in which it is not necessary to assume the completeness of X_1 and X_2 follows readily from the two lemmas on which the lattice theorem is based. (Received July 21, 1941.)

473. J. F. Paydon: *Convergence regions and value regions for continued fractions.* Preliminary report.

The author shows that if U is the circular region $|z| \leqq r$, $r \leqq 1/4$, and V is the circular region $|z-\alpha| \leqq \beta$ where $\alpha=[1+2r-(1-4r)^{1/2}]/(4r+2r^2)$, $\beta=[1-r-(1+r)(1-4r)^{1/2}]/(4r+2r^2)$, and if $a_2, a_3, \cdots$ lie in U, then the continued fraction $f=1/1+a_2/1+a_3/1+\cdots$, which is necessarily convergent, has its value in V. Moreover if v is on the boundary of V, then there is one and only one continued fraction with elements in U whose value is v. This uniqueness property is also established for the parabolic element region and companion value region of Scott and Wall (abstract 46-11-503). The following convergence theorem is also proved: If the elements of f lie in the region $R(z) \geqq 0$, $I(z) \geqq 0$, $|z| - R(z) \leqq 1$, then f converges if and only if (a) some a_n is 0, or (b) $a_n \neq 0$, $(n=2, 3, 4, \cdots)$, and $\sum |b_n|$ diverges where, $b_1=1$, $a_n=1/b_{n-1}b_n$, $(n=2, 3, 4, \cdots)$. (Received August 7, 1941.)

474. Maxwell Reade and E. F. Beckenbach: *Square averages and a class of harmonic polynomials.*

Harmonic functions are characterized by the property that they are identical with their own circular averages, both areal and peripheral. The authors delineate the classes of functions which are identical with their own: (a) square averages, (b) rectangular averages. The first class is a class of harmonic polynomials, while the latter class is a more general class of polynomials. A typical result is the following: If the function $f(x, y)$ is superficially summable over a finite simply connected domain D, then a necessary and sufficient condition that $f(x, y)=(1/4h^2)\int_{-h}^{h}\int_{-h}^{h} f(x+\xi, y+\eta)d\xi d\eta$ hold for each point (x, y) in D, for all h sufficiently small, is that $f(x, y)$ be a harmonic polynomial of the form $f(x, y)=A_0+A_1x+A_2y+A_3xy+A_4(x^2-y^2)+A_5(x^2-3xy^2)+A_6(3x^2y-y^3)+A_7(x^3y-y^3x)$, where the A_i are constants. (Received August 27, 1941.)

475. J. F. Ritt: *Complete difference ideals.*

This paper appeared in full in the American Journal of Mathematics for October, 1941. (Received September 19, 1941.)

476. Rubin Schatten: *On the direct product of Banach spaces.*

Two Banach spaces E_1 and E_2 may be combined in two ways: the well known "sum" $E_1 \oplus E_2$, and $E_1 \otimes E_2$, where $\otimes$ refers to the distributive operation. The present paper deals with the second construction in its algebraic aspects, presents a theory of possible crossnorms, and considers the conjugate spaces and their crossnorms. The existence of a greatest crossnorm and a least crossnorm whose conjugate is also a crossnorm has been proved, and that they are conjugate to each other. A self-conjugate crossnorm has been constructed which is a generalization of the norm for Hilbert spaces given by F. J. Murray and John von Neumann (Annals of Mathematics, (2), vol. 37 (1936), p. 127). (Received September 26, 1941.)

477. H. S. Wall: *The behavior of certain Stieltjes continued fractions near the singular line.*

The author shows that the function $f(z)$ represented by a continued fraction of the form $1/1+g_1z/1+(1-g_1)g_2z/1+(1-g_2)g_3z/1+\cdots$ in which $0<g_n<1$, $n=1, 2, 3, \cdots$, and the series $\sum|1-2g_n|$ converges, approaches a finite limit $\alpha(s)$ as $z\to -s$, $s>1$, along any path in the upper half-plane, and the limit $\bar{\alpha}(s)$, the complex conjugate of $\alpha(s)$, as $z\to -s$ from the lower half-plane. The function $\alpha(s)$ is continuous and not real, $s>1$. (Received August 18, 1941.)

Applied Mathematics

478. Arthur Korn: *On the problem of pulsating spheres in a liquid.*

The problem of the pulsating spheres treated for the first time almost 80 years ago by the Norwegian mathematician C. A. Bjerknes and chosen by the author as the point of departure for a mechanical theory of gravitation and the electromagnetic field, must be considered as a fundamental problem of mathematical physics. Interactions inversely proportional to the square of the distance can solely be produced by such spheres changing their volumes periodically with approximately the same frequency in an incompressible or approximately incompressible fluid. The sign of the forces of interaction (attraction in the case of equal phases, repulsion in the case of opposite phases) is a certain difficulty for the hypothesis that electrical particles are pulsating spheres. The sign can be changed, if one imposes on the spheres the condition that they maintain the amplitudes of their pulsations at any displacement of the spheres, a condition which is not imposed in Bjerknes' problem. The calculations are a little complicated, if one has to find out the pressure at the surfaces of the spheres and if one derives from these pressures the forces in question. Here is given a rather simple way of deriving the forces from the energy of the liquid in the case of displacements of the spheres, and one can easily understand the change of the sign, if one imposes on the spheres the condition that they maintain the amplitudes of their pulsations at any displacement. (Received September 22, 1941.)

Geometry

479. George Comenetz: *Isotropic curves on a surface.*

Let S be a regular analytic surface in complex three-dimensional euclidean space, and O a point of S at which the tangent plane T is isotropic. Such a point is singular for the differential equation $ds^2=0$ of the isotropic net of curves on S. The problem is to describe the regular analytic isotropic curves on S that pass through O. Let L be the isotropic straight line in T through O, and p its order of contact with S. Draw an arbitrary curve F on S having contact of order p with L at O. Draw the normal to S at a point V of F, and let q be the "order of coincidence" of the normal with its limit L, as V approaches O. Then if $q>p/2$ the number of isotropic curves is 0 or 2, if $q=p/2$ it is 1 or 2 or $1+\infty^1$, and if $q<p/2$ it is 2. The conditions under which the different possibilities are realized, and the orders of contact of the curves with L and one another, are stated. (Received September 15, 1941.)

480. J. J. DeCicco: *Bi-isothermal systems of curves.*

A generalization of the concept of isothermal family of curves in four-dimensional space S_4 is obtained in this paper. Kasner has termed the correspondences of S_4, de-

fined by pairs of functions of two complex variables, the pseudo-conformal group G. This is characterized by the preservation of Kasner's pseudo-angle between a curve and a hypersurface at their common point. Any system of ∞^3 curves which is pseudo-conformally equivalent to a parallel set of lines is said to be bi-isothermal. Any such system consists of ∞^2 isothermal families, each family living on a conformal surface. A characteristic property of bi-isothermal systems is that the pseudo-angle between this system and any parallel pencil of hyperplanes is a biharmonic function. Finally G is characterized within the group of point transformations by the preservation of the totality of all bi-isothermal systems. (Received September 29, 1941.)

481. Walter Prenowitz: *Descriptive geometries as multigroups.*

Let G be the set of points of a descriptive space of arbitrary dimension. In G, define $a+b$ as the set of points between a and b if $a \neq b$, and $a+a$ as a. Then G becomes an abelian multigroup of special type. Convex sets and linear manifolds appear respectively as semigroups (subsets closed under $+$) and subgroups of G. Half-spaces (rays, half-planes, and so on) are cosets, when the latter are properly defined, and some of the elementary properties of half-spaces follow from a coset decomposition theorem. Elementary properties of these three types of figures are derived algebraically and many familiar group theoretic concepts as factor group, homomorphism, congruence relation are used in the development. (Received September 29, 1941.)

482. C. V. Robinson: *Spherical theorems of Helly type and congruence indices of spherical caps.*

The theorems mentioned in an earlier abstract (47-1-67) have been extended to the n-sphere. The principal theorem of Helly type reads: If each $n+k+2$ members of a family of convex subsets of the n-sphere intersect and if one member contains no k-dimensional great hypersphere then there is a point common to all the sets of the family. A study is also made of the congruence indices of spherical caps of various radii with respect to the class of semi-metric spaces. For example, it is found that a cap of spherical radius $\rho < \pi r/2$ of the 2-sphere of radius r will contain isometrically any semi-metric space of more than 6 points if every quadruple of the space is isometrically imbeddable in the cap, that is, the cap has the congruence indices [4, 2]. (Received September 23, 1941.)

Statistics and Probability

483. J. F. Daly: *A problem in estimation.*

Consider a normal population in which each individual is characterized by the variates $y_1, \cdots, y_p, y_{p+1}, y_{p+2}$. Suppose that the latter two are not directly observable, but that for given values of y_{p+1}, y_{p+2} the first set of y's is independently distributed about the "regression line" $y_k = y_{p+1} + k y_{p+2}$ $(k = 1, \cdots, p)$ with a common variance σ^2. For each individual, one can thus determine values $\hat{y}_{p+1}$, $\hat{y}_{p+2}$ from the observed $y_1, \cdots, y_p$, using the method of least squares. Assuming a similar relation between the expected values of $y_1, \cdots, y_{p+2}$ in the original population, these estimates $\hat{y}_{p+1}$, $\hat{y}_{p+2}$ are of course unbiased. However, if we calculate these $\hat{y}$'s for each individual of a sample of N and substitute them in the Pearson product-moment correlation formula the estimate of the correlation between y_{p+1} and y_{p+2} thus obtained is somewhat biased. The bias depends on the number of observable y's and on the size of the variances and covariances of y_{p+1}, y_{p+2} relative to σ^2. (Received September 2, 1941.)

484. Hilda P. Geiringer: *Some observations on analysis of variance theory.*

The distribution of the test functions used in the analysis of variance has been determined by R. A. Fisher for mutually independent chance variables subject to the same normal law. If it is not possible to determine the distributions of test functions for small samples and sufficiently general populations, the expectations and the variances of the test functions computed under appropriately general assumptions may be used. The expectations of the two quadratic forms known as "variance within" and "among" classes are equal even for non-normal but equal populations (Bernoulli series). Moreover it can be proved that the expectation of their quotient equals one. In case of Lexis and Poisson series, certain inequalities are investigated. These different criteria are completed by the computation of the variances. Also for some other symmetrical test functions general populations may be assumed. The Lexis as well as the Poisson series may then be characterized by equalities. Finally, omitting the restriction to independent chance variables, different kinds of mutual dependence are studied. These investigations which lead to new inequalities among the expectations seem to be related to Fisher's intraclass correlation and to supplement this idea. (Received August 11, 1941.)

485. E. J. Gumbel: *Simple tests for given statistical hypotheses.*

The probability integral transformation reduces the comparison of an observed distribution with a theoretical continuous distribution to the comparison of n observed points (n being the number of observations) with an equidistribution over the interval 0, 1. The well known shortcomings of the χ^2 test, the dependency on the classification and the necessity to combine the contents of the extreme cells can thus be avoided. The comparison of the observed and the theoretical means and measures of dispersion is very simple. The theoretical cumulative frequency of the mth point is the straight line $\bar{y}=m:(n+1)$ where $\bar{y}$, the mean position of the mth point, is plotted as the abscissa and $m(=1, 2, \cdots, n)$, the serial number, as the ordinate. The observed points will be scattered about this straight line. To judge the significance of these deviations, two control curves are constructed: To each point on the straight line correspond points $(\bar{y}-\sigma, m)$ and $(\bar{y}+\sigma, m)$, where σ the standard deviation of the mth point is given by $\sigma(n+2)^{1/2}=(\bar{y}(1-\bar{y}))^{1/2}$. (Received September 27, 1941.)

486. Abraham Wald: *Large sample distribution of the likelihood ratio.*

The large sample distribution of the likelihood ratio has been derived by S. S. Wilks (Annals of Mathematical Statistics, vol. 9 (1938)) in the case of a linear composite hypothesis and under the assumption that the hypothesis to be tested is true. Here a general composite hypothesis is considered and the distribution in question is derived also in case that the hypothesis to be tested is not true. Let $f(x_1, \cdots, x_p, \theta_1, \cdots, \theta_k)$ be the joint probability density function of the variates $x_1, \cdots, x_p$ involving k unknown parameters $\theta_1, \cdots, \theta_k$. Denote by H_ω the hypothesis that the true parameter point $\theta=(\theta_1, \cdots, \theta_k)$ satisfies the equations $\xi_1(\theta)=\cdots=\xi_r(\theta)=0$ $(r\leq k)$. Denote by λ_n the likelihood ratio statistic for testing H_ω on the basis of n independent observations on $x_1, \cdots, x_p$. It is shown that under certain restrictions on $f(x_1, \cdots, x_p, \theta)$, $\xi_1(\theta), \cdots, \xi_r(\theta)$ one obtains $\lim_{n=\infty}\{P(-2\log\lambda_n<t|\theta)-P[Q_n(\theta)<t|\theta]\}=0$ uniformly in t and θ, where $Q_n(\theta)$ denotes a certain quadratic

form of r normally distributed variates, and $P(z<t|\theta)$ denotes the probability that z is less than t calculated under the assumption that θ is true. (Received August 11, 1941.)

TOPOLOGY

487. Samuel Eilenberg and Saunders MacLane: *Infinite cycles and homologies.*

An abelian group E is an extension of G by H if G is a subgroup of E and H is the corresponding factor group. With a suitable definition of equivalence and addition the extensions of G by H form a group. The authors consider the qth homology group of an abstract complex with coefficients in a group G obtained using infinite cycles and purely formal boundaries. A direct sum decomposition of this group is given, the first member of which is isomorphic with the group of all homorphic mappings of the qth integral cohomology group (finite chains and cocycles) into G and the second with the group of all abelian group extensions of G by the $(q+1)$th cohomology group. Various consequences of this result are obtained. (Received September 4, 1941.)

488. R. L. Moore: *On continua with dendratomic subsets.*

It is shown that in order that a compact continuum should have dendratomic subsets it is necessary and sufficient that it should not be a web. It follows that every compact continuum which is not a triod has such subsets. In particular every compact irreducible continuum between two points has them. (Received September 5, 1941.)

489. N. E. Steenrod: *Topological methods for the construction of tensor functions.*

The space M' of point tensors (fixed order and weight) over a differentiable manifold M of class r is shown to be a differentiable manifold of class $r-1$ forming a fibre bundle over M in the sense of Whitney. If M'' is a submanifold of M' which is still a fibre bundle over M with fibres F'', methods are given for attempting to ascertain if a tensor function of class $r-1$ exists with values in M''. An approximation theorem reduces the problem to finding a function which is merely continuous. If h is the smallest integer such that the homotopy group $\pi_h(F'')\neq 0$, a function of the required kind may be defined over the h-dimensional skeleton M^h of M. Any such determines a cocycle c^{h+1} in M with coefficients in the groups $\pi_h(F'')$. It is necessary to use here a homology theory based on local coefficient groups connected by local isomorphisms. In order that a tensor of the required kind exist over M^{h+1}, it is necessary and sufficient that $c^{h+1}\backsim 0$. The existence and properties of the characteristic cohomology class are also proved for the more general type of fibre space introduced by Hurewicz and the author. (Received August 5, 1941.)

490. Henry Wallman: *Dimension for general spaces.*

For separable metric spaces there is a body of theorems of deep geometric appeal centering around the concept of dimension. In line with recent trends it seems natural to inquire whether one can extend the domain of application of dimension to spaces of more general character. The three common dimension functions are the Menger-Urysohn dimension d_1, the Čech dimension-in-the-large d_2 and the Lebesgue covering dimension d_3. It turns out that although there is not the slightest difficulty in applying these dimension-functions to spaces of the most general sort, one can hardly

proceed thereafter to build up a dimension theory for such general spaces with these for: (a) there exists a countable set for which d_1 is positive, (b) neither d_2 nor d_3 is monotonic ($A \subset B$ does not imply $d_2(A) \leq d_2(B)$ nor $d_3(A) \leq d_3(B)$) even for subsets of so regular a space as a bicompact Hausdorff space, (c) d_1, d_2, d_3, do not coincide. (Upon the coincidence of d_1, d_2, and d_3 for separable metric spaces rests a very great deal of the power of the dimension concept.) (Received September 3, 1941.)

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